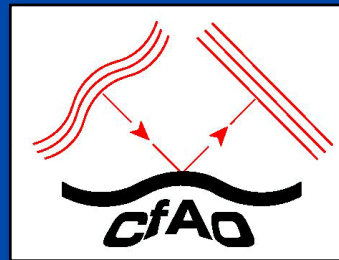


# *Lecture 14*

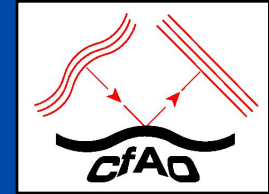
## *AO System Optimization*



Claire Max  
Astro 289, UC Santa Cruz  
March 1, 2016

# Optimization of AO systems

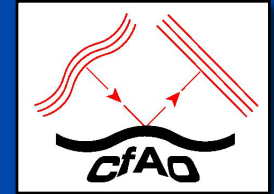
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- If you are designing a new AO system:
  - How many actuators?
  - What kind of deformable mirror?
  - What type of wavefront sensor?
  - How fast a sampling rate and control bandwidth (peak capacity)?
- If you are using an existing AO system:
  - How long should you integrate on the wavefront sensor?  
How fast should the control loop run?
  - Is it better to use a bright guide star far away, or a dimmer star close by?
  - What wavelength should you use to observe?

# *Issues for designer of astronomical AO systems*

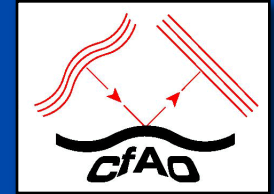
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- **Performance goals:**
  - Sky coverage fraction, observing wavelength, degree of image compensation needed for science program (Strehl ratio or contrast ratio, ...), field of view
- **Parameters of the observatory:**
  - Turbulence characteristics (mean and variability), telescope and instrument optical errors, laser guide stars availability
- **AO parameters chosen in the design phase:**
  - Number of actuators, type of DM and WFS, sample rate, servo bandwidth, laser characteristics
- **AO parameters adjusted by user:**
  - Observing wavelength, guide star mag. & offset

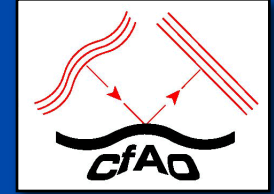
## *Example: Keck Observatory AO “Blue Book”*

---



- Made scientific case for Keck adaptive optics system
- Laid out the technical tradeoffs
- Presented performance estimates for realistic conditions
- First draft of design requirements

**The basis for obtaining funding commitment  
from the user community and observatory**



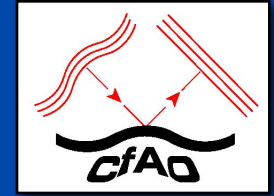
## *What is in the Keck AO Blue Book?*

---

- **Chapter titles:**
  1. Introduction
  2. Scientific Rationale and Objectives
  3. Characteristics of Sky, Atmosphere, and Telescope
  4. Limitations and Expected Performance of Adaptive Optics at Keck
  5. Facility Design Requirements
- **Appendices: Technical details and overall error budget**

## *Other telescope projects have similar “Books”*

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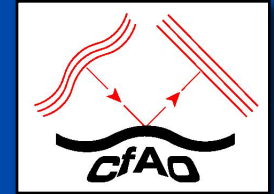


- Keck Telescope (10 m):
  - Had a “Blue Book” for the *telescope* itself
- Thirty Meter Telescope:
  - Series of design documents: Detailed Science Case, Science Based Requirements Document, Observatory Requirements Document, Operations Requirements Document, etc.

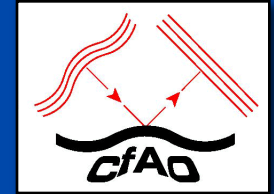
**These documents are the kick-off point  
for work on the “Preliminary Design”**

# *First, let's look at individual terms in error budget one by one*

---



- **Error budget terms**
  - Fitting error
  - WFS measurement error
  - Anisoplanatism
  - Temporal error
  
- **Figures of merit**
  - Strehl ratio (imaging)
  - FWHM (imaging and spectroscopy)
  - Encircled energy (spectroscopy)
  - Contrast ratio (high-contrast such as exoplanets)



## Fitting error: dependence of Strehl on $\lambda$ and DM degrees of freedom

Deformable mirror fitting error (continuous face sheet)

$$S = \exp(-\sigma_\phi^2) = \exp\left[-0.28\left(\frac{d}{r_0}\right)^{5/3}\right]$$

$$r_0(\lambda) = r_0(\lambda = 0.5\mu m)\left(\frac{\lambda}{0.5\mu m}\right)^{6/5}$$

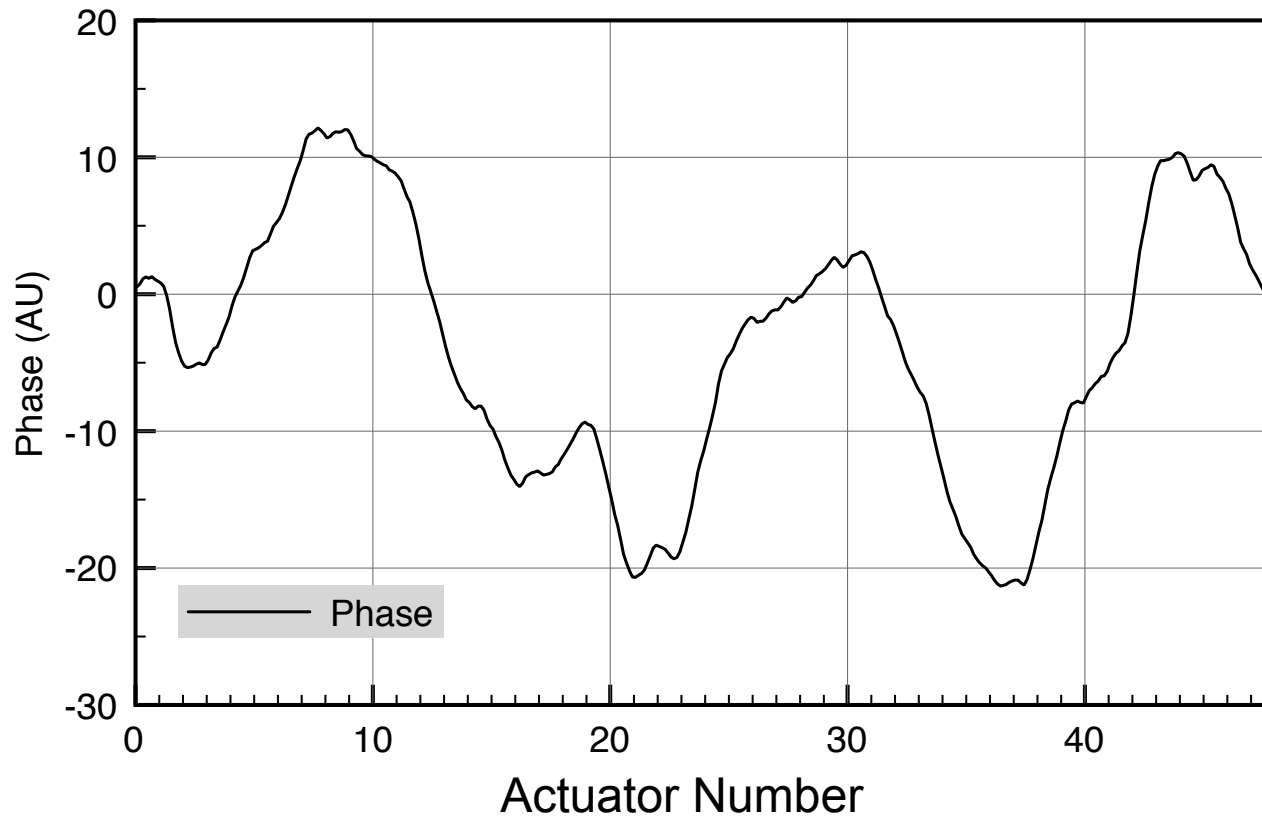
$$S = \exp\left[-0.28\left(\frac{d}{r_0(\lambda = 0.5\mu m)}\right)^{5/3}\left(\frac{0.5\mu m}{\lambda}\right)^2\right]$$

- Assume very bright natural guide star
- No meas't error or anisoplanatism or bandwidth error

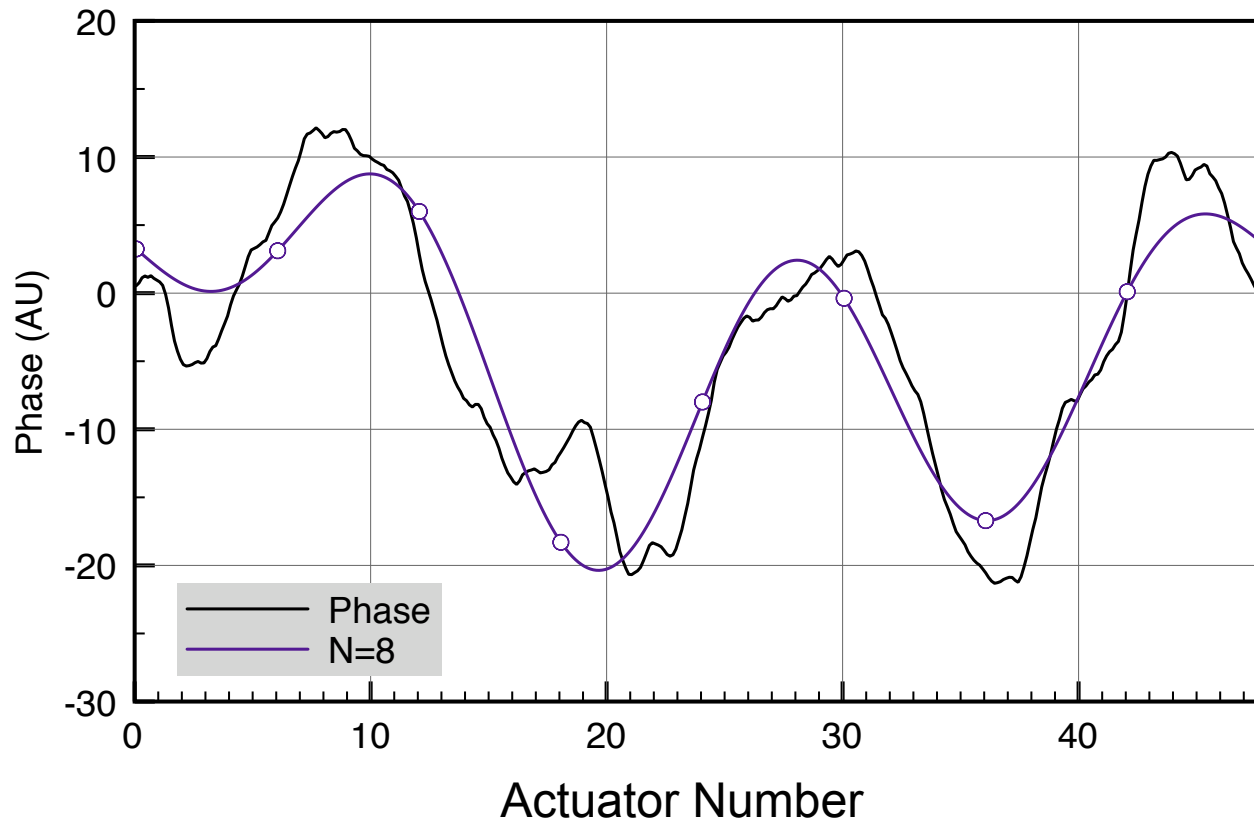
**Strehl increases for smaller subapertures  $d$  and longer observing wavelengths  $\lambda$**



Fitting a phase shape

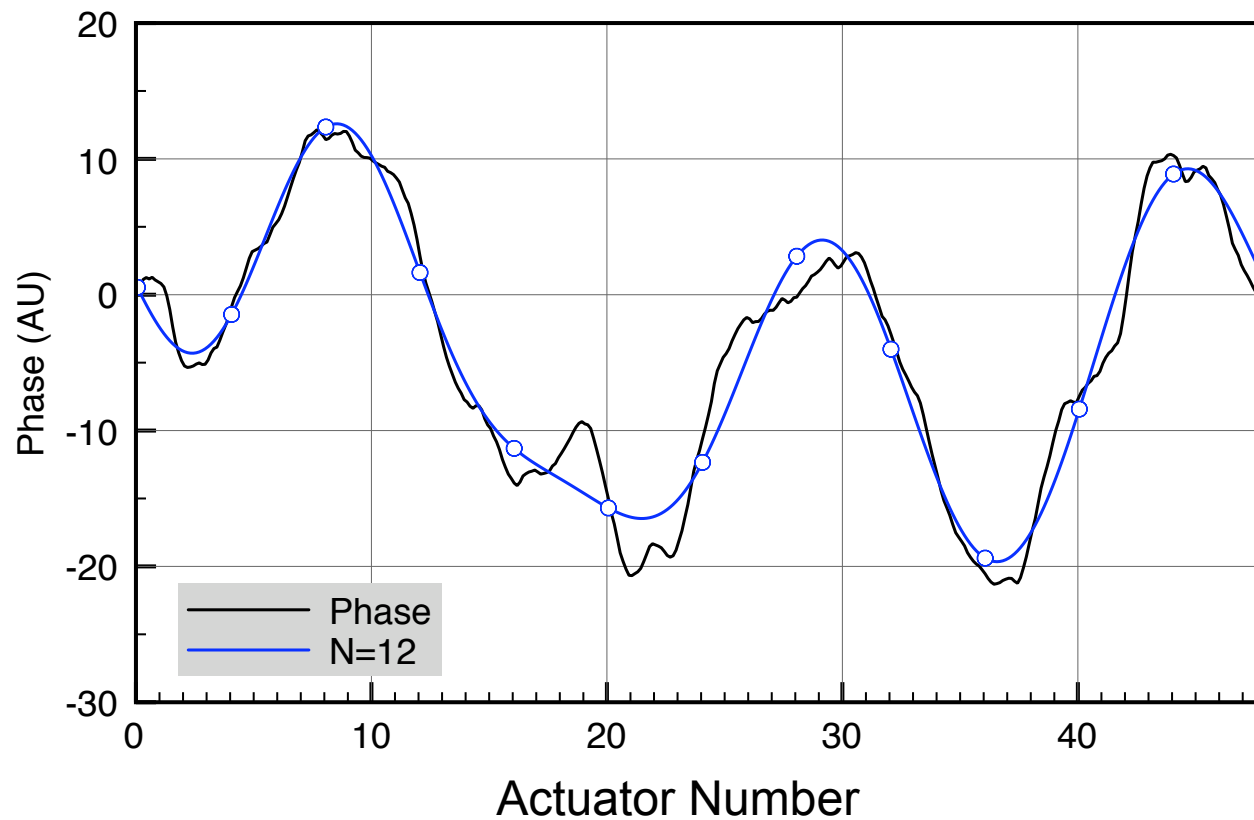


Fitting a phase shape



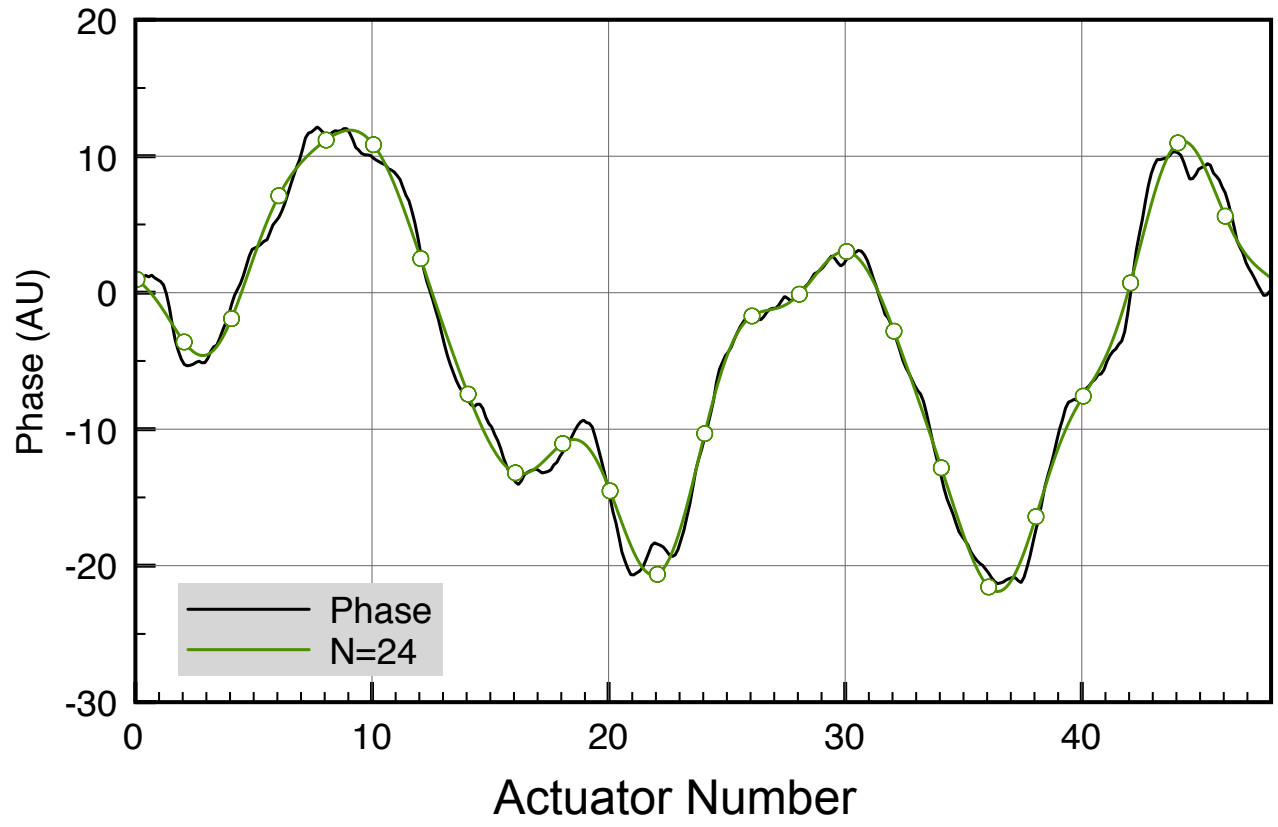
**More actuators = a better fit**

Fitting a phase shape



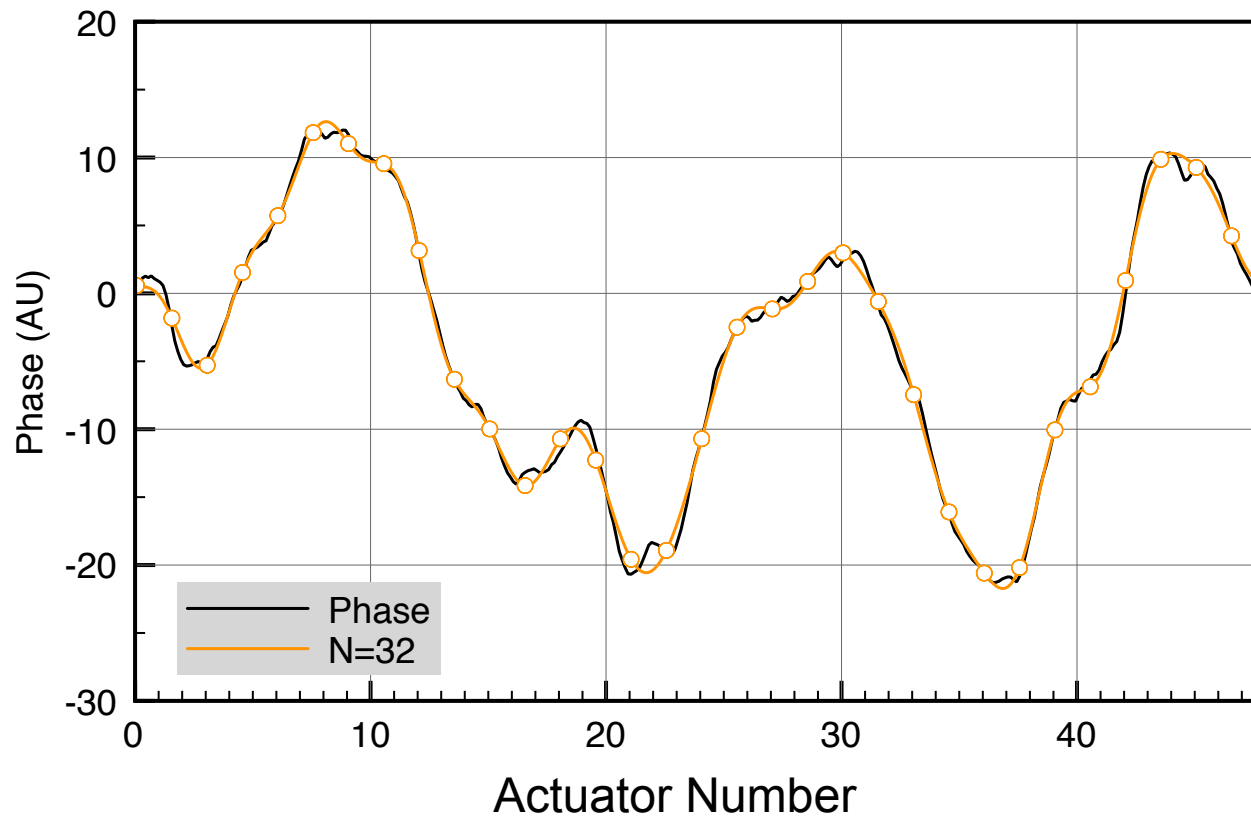
**More actuators = a better fit**

Fitting a phase shape



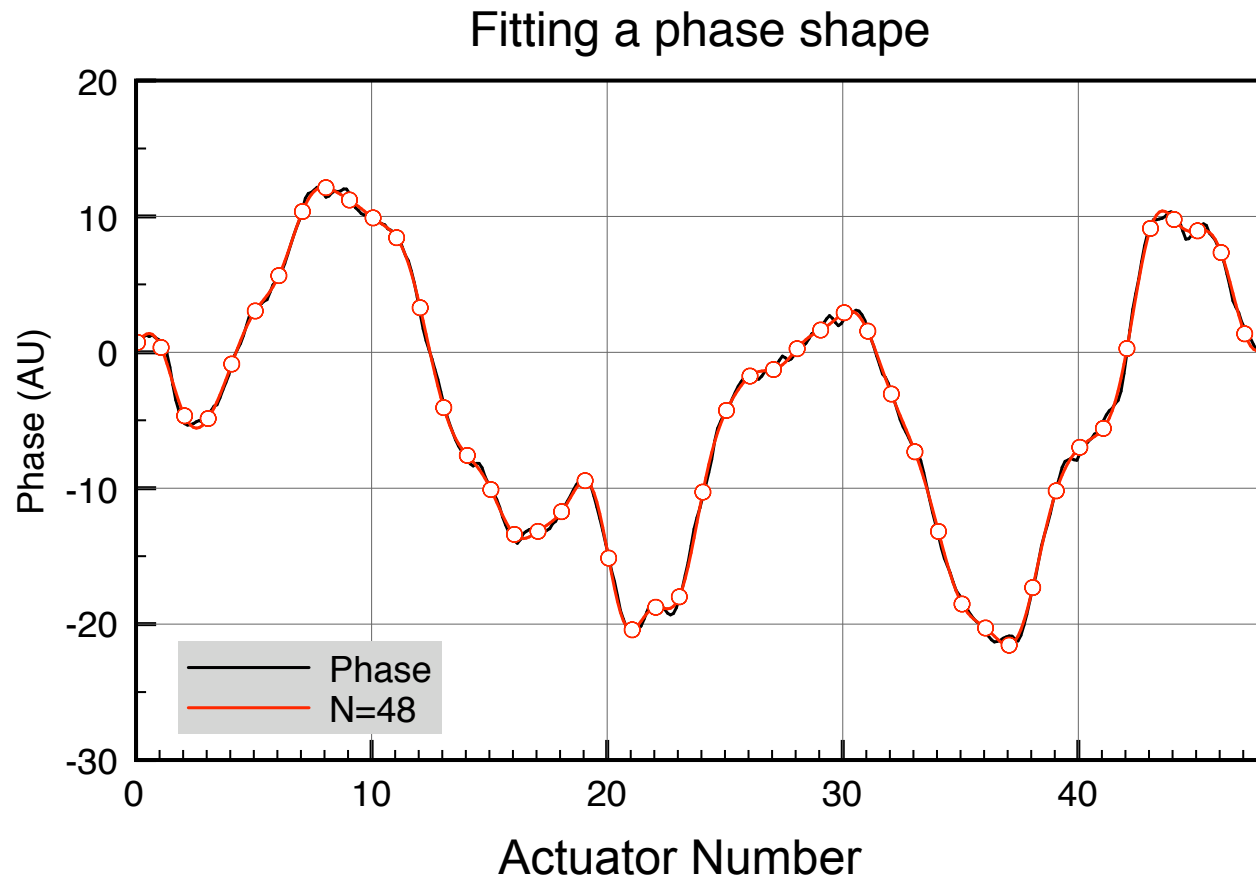
**More actuators = a better fit**

Fitting a phase shape



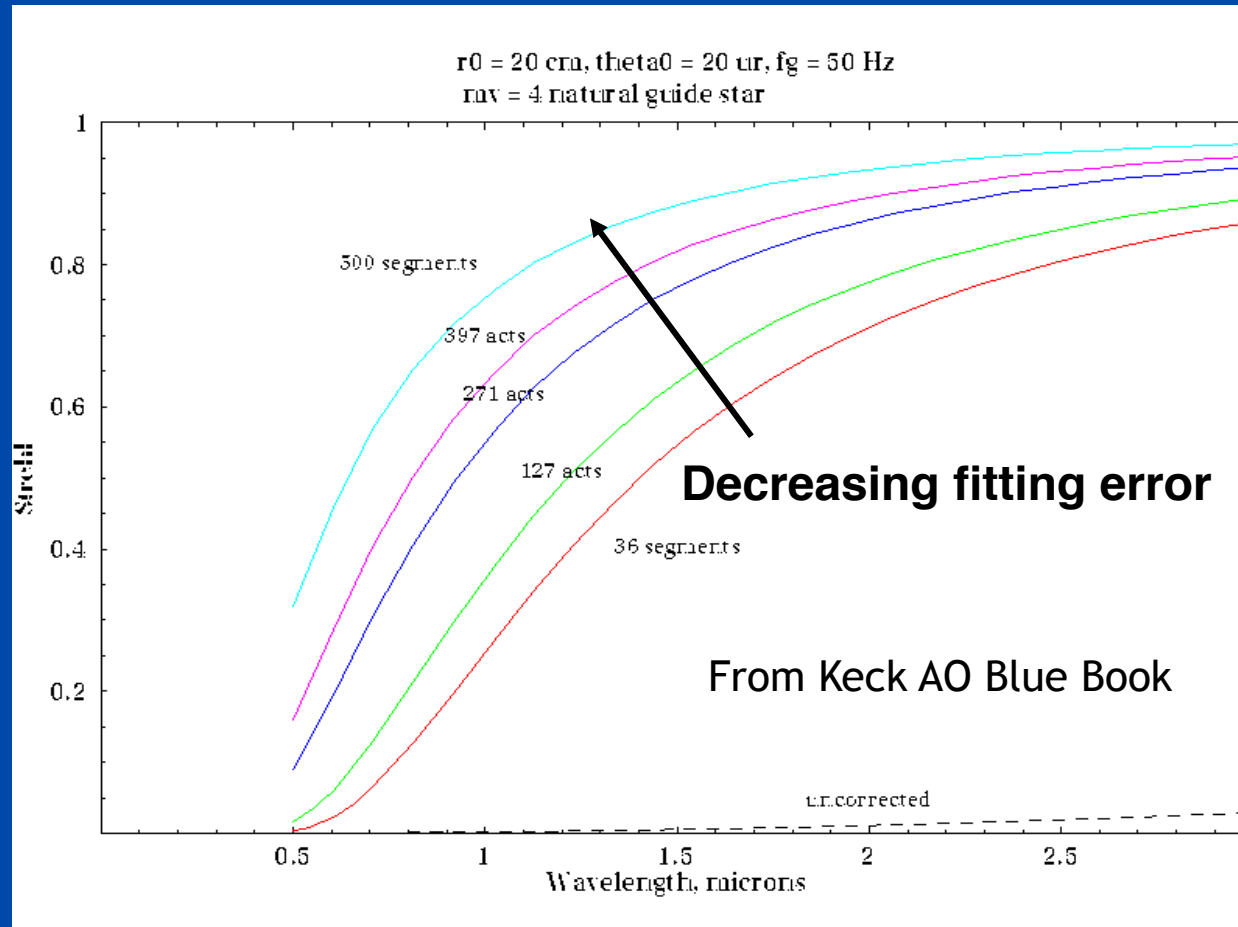
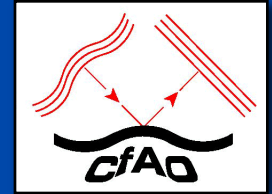
**More actuators = a better fit**

Thanks to Lisa Poyneer for these graphics



**More actuators = a better fit**

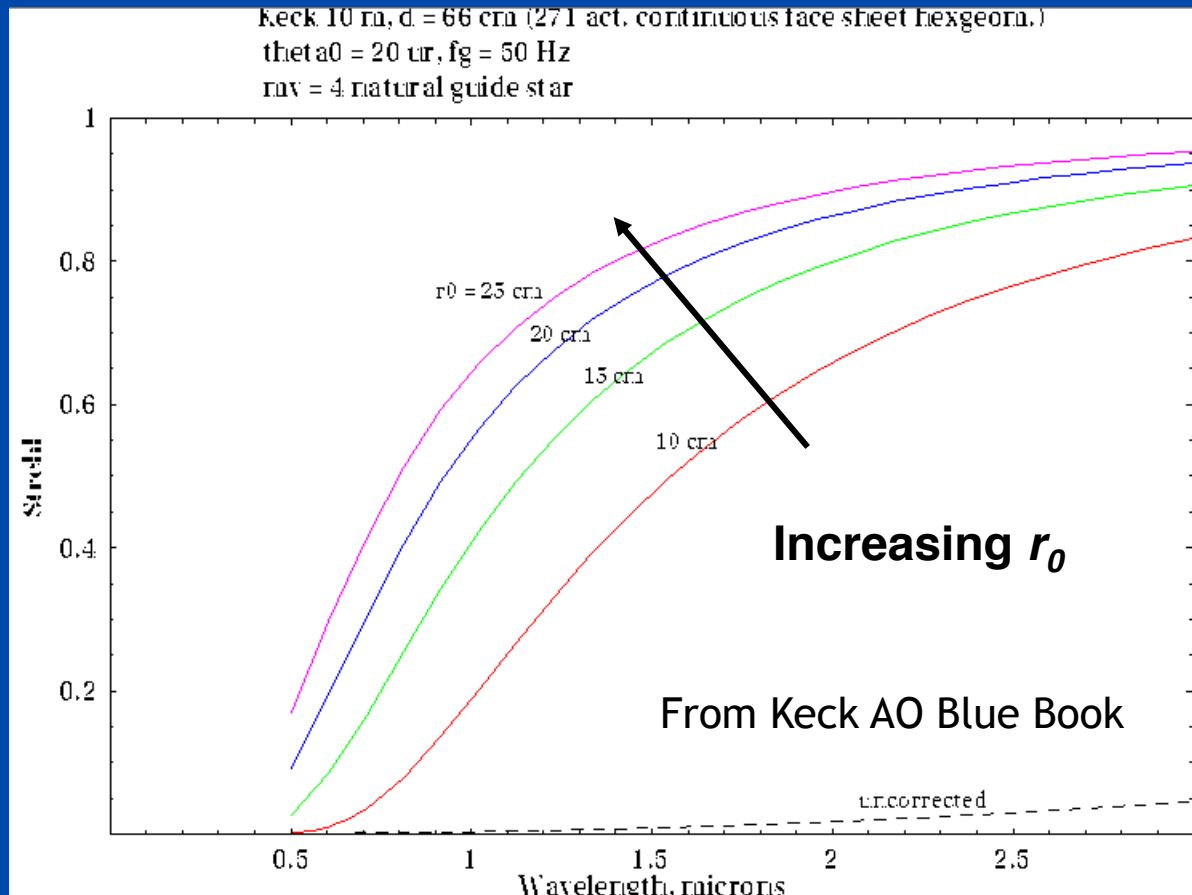
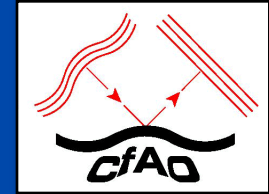
# Strehl increases for smaller subapertures and longer observing wavelengths



- Assume very bright natural guide star
- No meas't error or anisoplanatism or bandwidth error

Deformable mirror fitting error only

# Strehl increases for longer $\lambda$ and better seeing (larger $r_0$ )

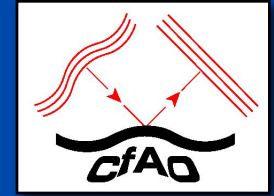


- Assume very bright natural guide star
- No meas't error or anisoplanatism or bandwidth error

Deformable mirror fitting error only



# Wavefront sensor measurement error: Strehl vs $\lambda$ and guide star magnitude



Assume no DM fitting error or other error terms

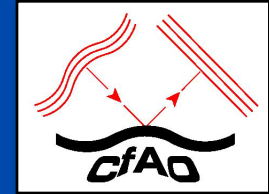
$$\sigma_{S-H}^2 \approx \left( \frac{6.3}{SNR} \right)^2$$

$$S = \exp(-\sigma_{S-H}^2) = \exp\left[-\left(\frac{6.3}{SNR}\right)^2\right]$$

*SNR* increases as flux from guide star increases

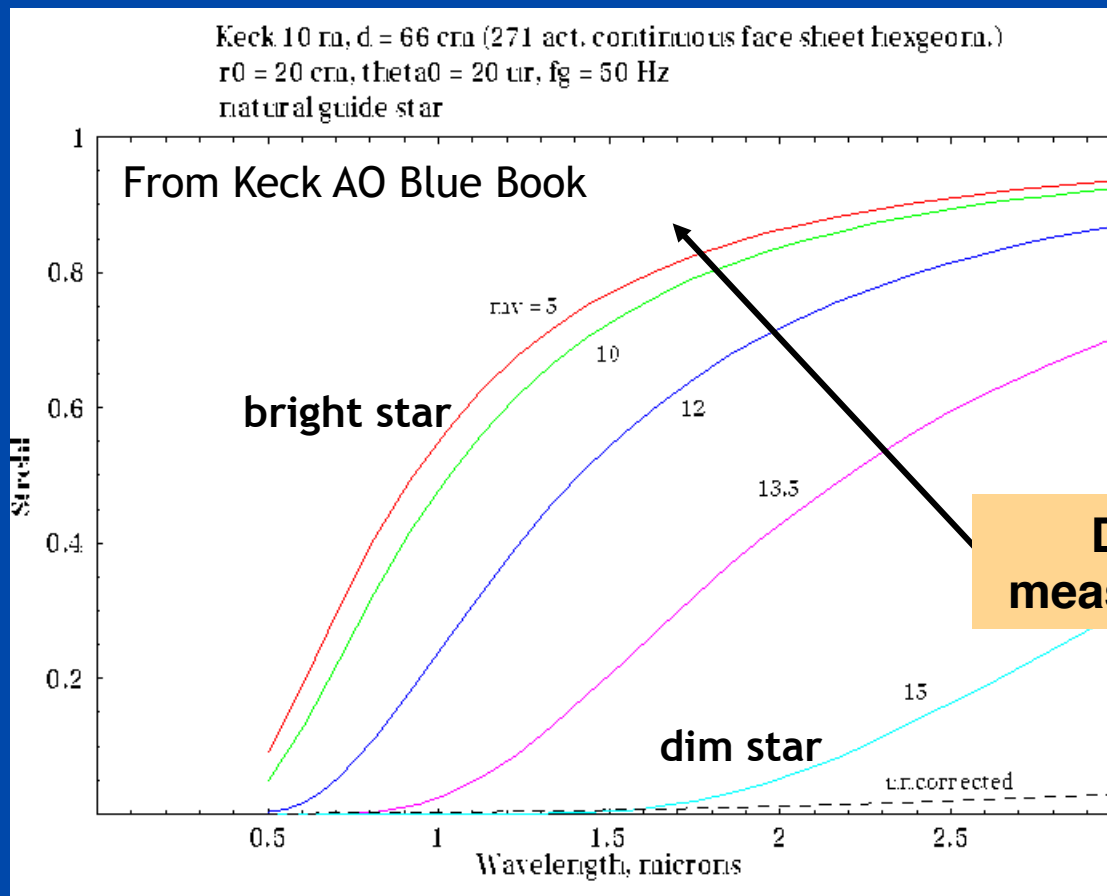
**Strehl increases for brighter guide stars**

But: SNR will decrease as you use more and more subapertures, because each one will gather less light



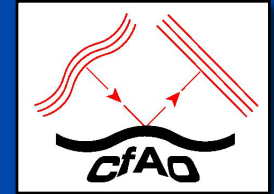
# WFS Measurement Error: Strehl increases for brighter guide stars

Assumes no DM fitting error or other error terms



Decreasing  
measurement error

# *Strehl vs $\lambda$ and guide star angular separation (anisoplanatism)*

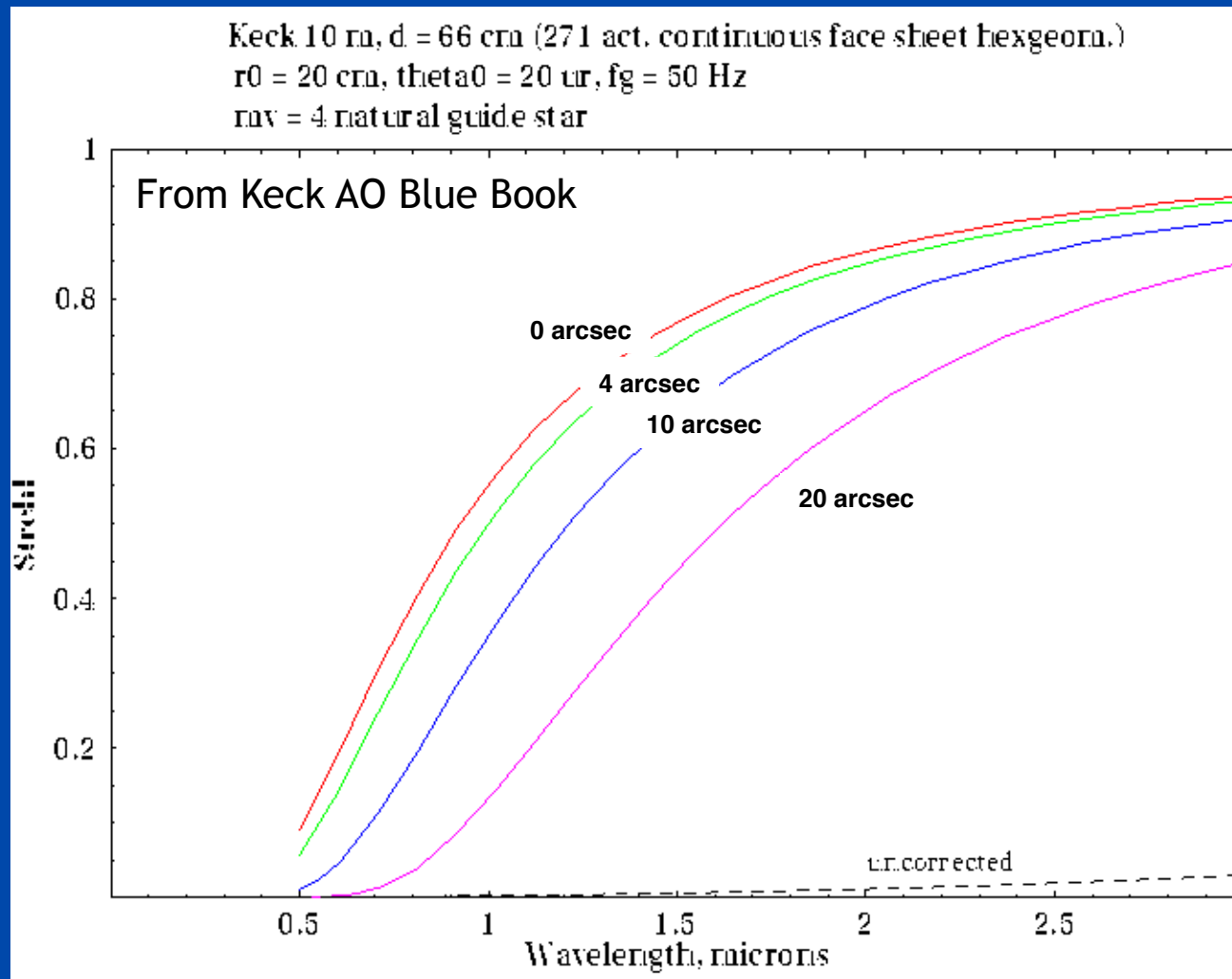
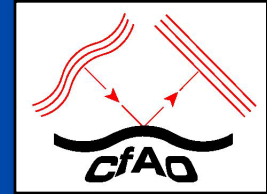


$$S = \exp[-\sigma_{iso}^2] = \exp\left[-\left(\frac{\theta}{\theta_0}\right)^{5/3}\right], \quad \theta_0 = \frac{r_0}{h} \propto \lambda^{6/5}$$

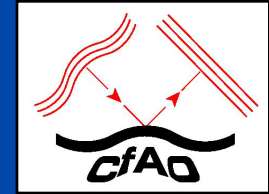
$$S = \exp\left[-\left(\frac{\theta}{\theta_0(0.5\mu m)}\right)^{5/3} \left(\frac{0.5\mu m}{\lambda}\right)^2\right]$$

**Strehl increases for smaller angular offsets  
and longer observing wavelengths**

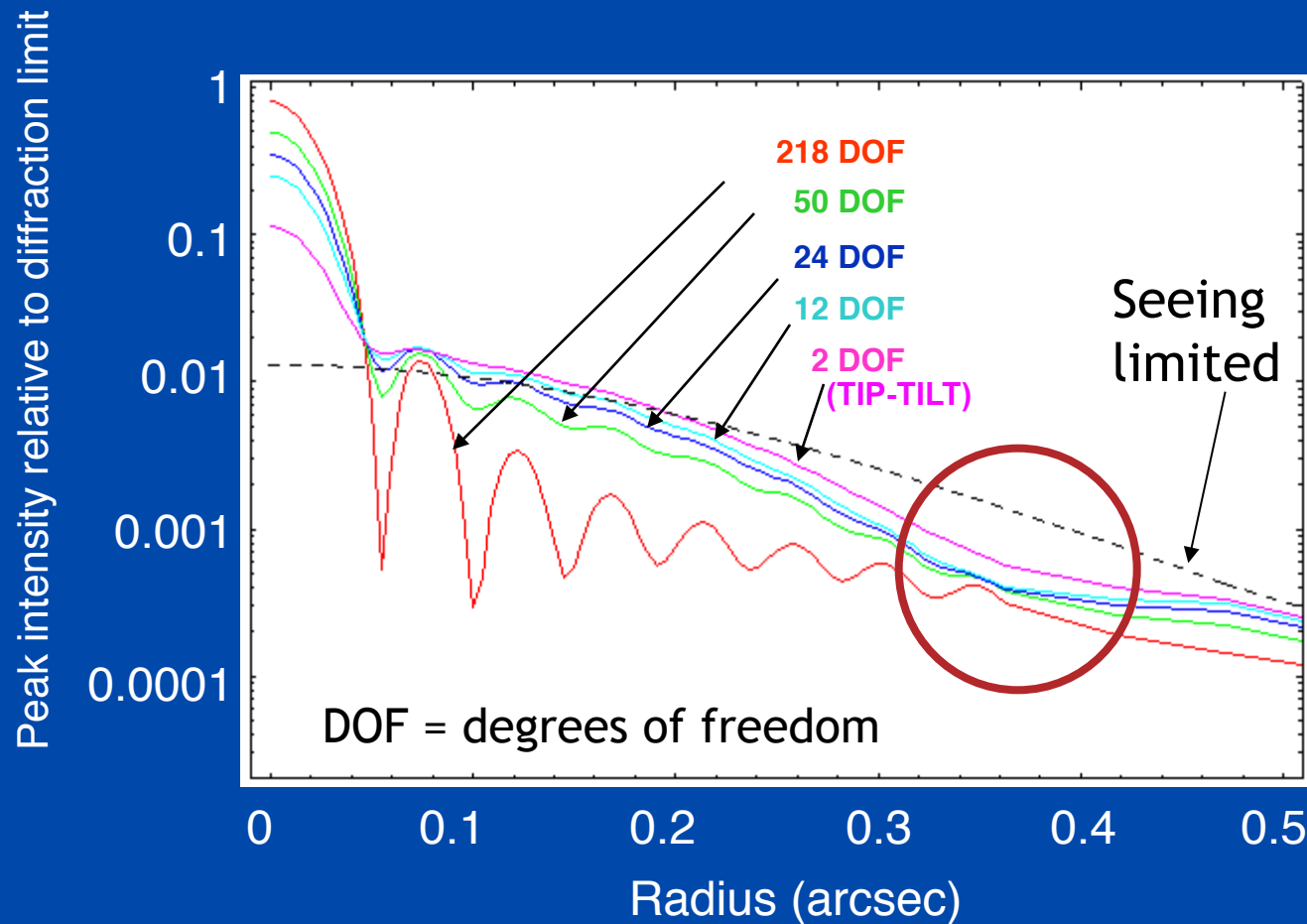
# Strehl increases for smaller angular offsets and longer observing wavelengths



# PSF with bright guide star: more degrees of freedom $\Rightarrow$ more energy in core

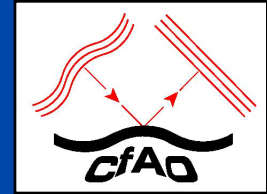


“Very bright guide star”: Assume no measurement error in WFS



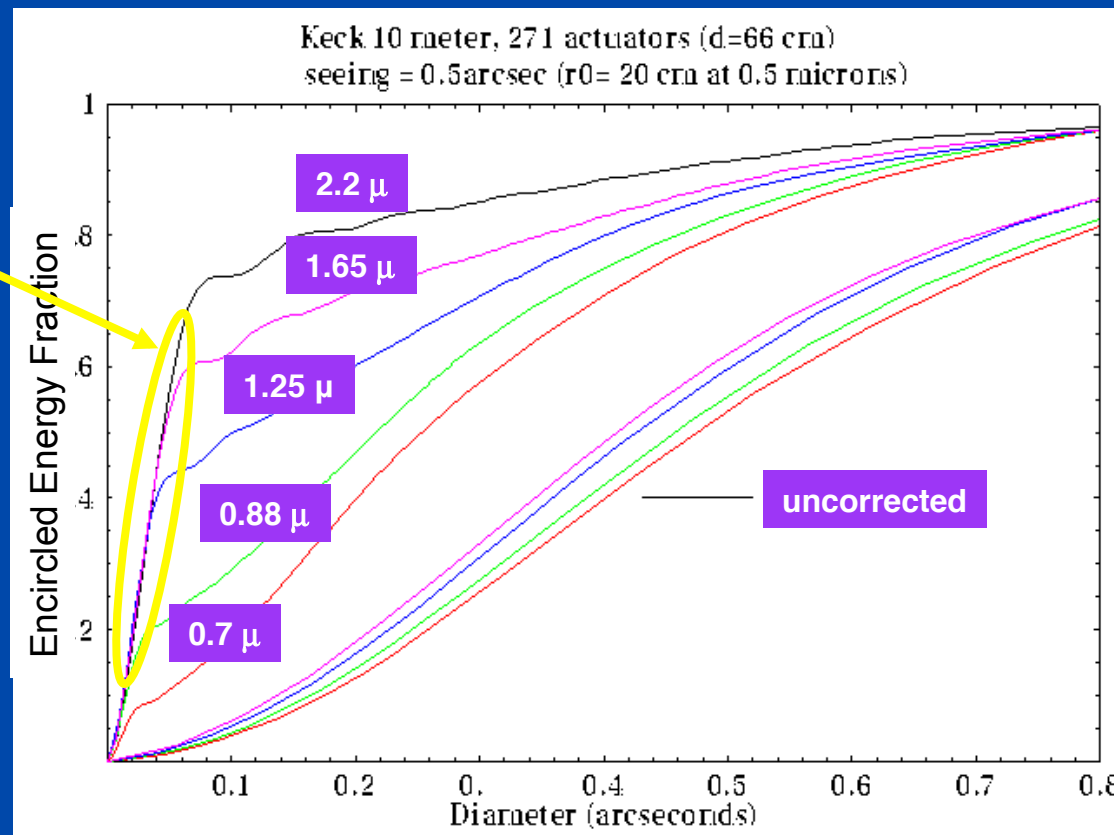
Point Spread Function very bright star,  $\lambda = 2.2 \mu\text{m}$ ,  $D / r_0 = 8.5$

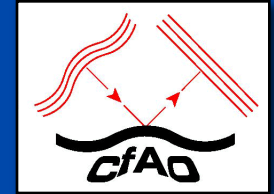
# What matters for spectroscopy is "Encircled Energy"



Fraction of light encircled within diameter of xx arc sec

Diffraction limited

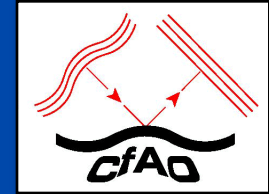




## *Overall system optimization*

---

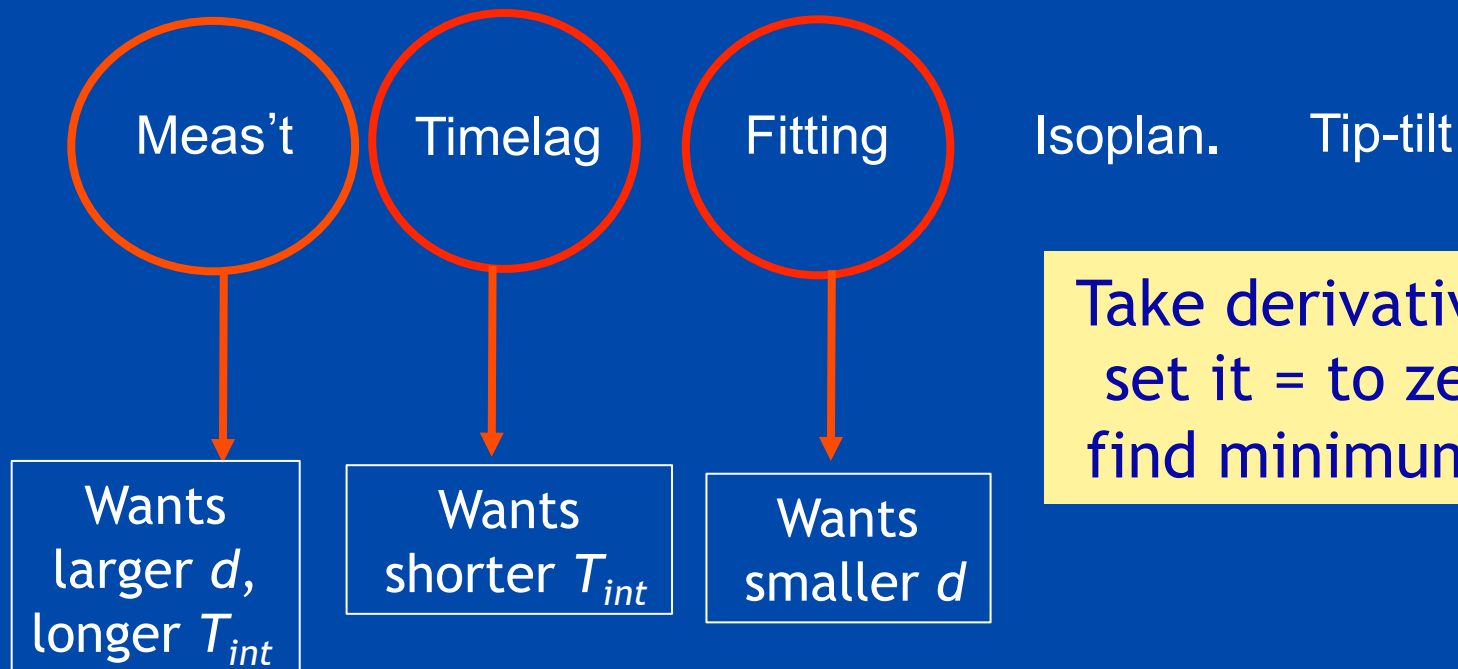
- Concept of error budget
  - Independent contributions to wavefront error from many sources
- Minimize overall error with respect to a parameter such as integration time or subaperture size



## Error model: mean square wavefront error is sum of squares of component errors

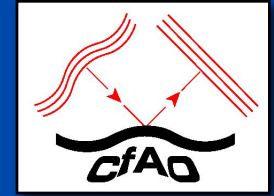
- Minimize the mean square error in wavefront phase

$$\sigma_p^2 = \sigma_{Meas}^2 + \sigma_{BW}^2 + \sigma_{DM}^2 + \sigma_{iso}^2 + \sigma_{T-T}^2 + \dots$$





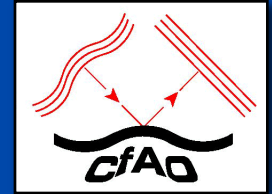
# Signal to Noise Ratio for a fast CCD detector



$$SNR = \frac{Flux \times T_{int}}{Noise} = \frac{Flux \times T_{int}}{\left[ \sigma_{PhotonNoise}^2 + \sigma_{SkyBkgnd}^2 + \sigma_{DarkCurrent}^2 + \sigma_{ReadNoise}^2 \right]^{1/2}}$$

- *Flux* is the average photon flux (detected photons/sec)
- $T_{int}$  is the integration time of the measurement,
- *Sky background* is due to OH lines and thermal emission
- *Dark current* is detector noise per sec even in absence of light (usually due to thermal effects)
- *Read noise* is due to the on-chip amplifier that reads out the charge after each exposure

## Short readout times needed for wavefront sensor $\Rightarrow$ read noise is usually dominant

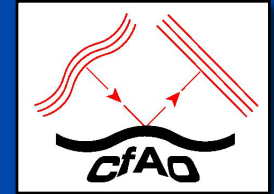


- *Read-noise dominated:* read noise  $\gg$  all other noise sources
- In this case SNR is

$$SNR_{RN} = \frac{Flux \times T_{int}}{\left[ R^2 n_{pix} \right]^{1/2}} = \frac{Flux \times T_{int}}{R \sqrt{n_{pix}}}$$

where  $T_{int}$  is the integration time,  $n_{pix}$  is the number of pixels in a subaperture,  $R$  is the read noise/px/frame

## Now, back to calculating measurement error for Shack-Hartmann sensor



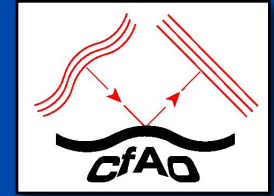
$$\sigma_{S-H}^2 \cong \left[ \frac{6.3}{SNR} \right]^2 \text{ rad}^2$$

- Assume the WFS is read-noise limited. Then

$$SNR_{S-H} = \frac{Flux \times T_{int}}{R \sqrt{n_{pix}}} \text{ and}$$

$$\sigma_{S-H}^2 = \left[ \frac{6.3}{SNR} \right]^2 = 6.3^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2}$$

## Error model: mean square wavefront error is sum of squares of component errors



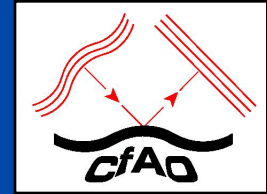
$$\sigma_p^2 = \sigma_{Meast}^2 + \sigma_{BW}^2 + \sigma_{DM}^2 + \sigma_{iso}^2 + \dots$$

$$\sigma_p^2 = 6.3^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \left( \frac{T_{control}}{\tau_0} \right)^{5/3} + \mu \left( \frac{d}{r_0} \right)^{5/3} + \left( \frac{\theta}{\theta_0} \right)^{5/3} + \dots$$

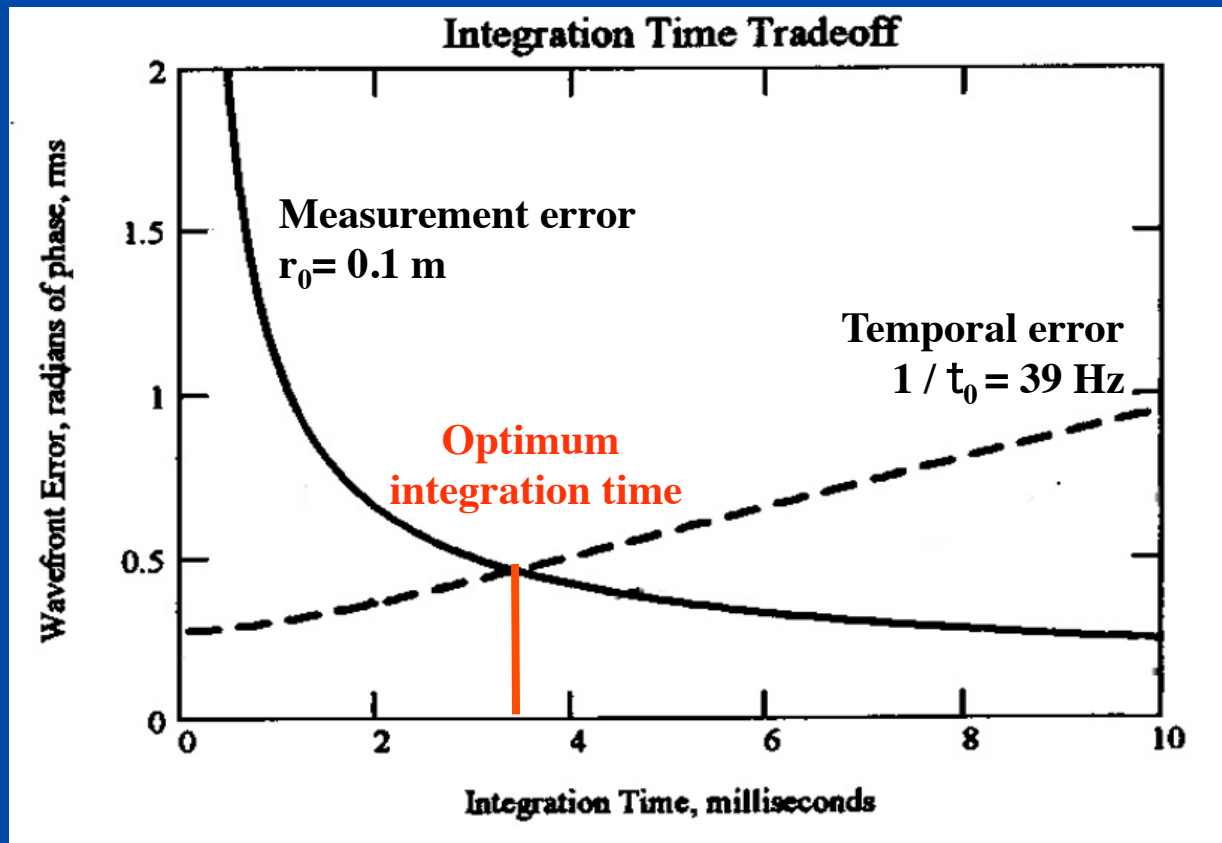
$T_{control}$  is the closed-loop control timescale, typically ~ 10 times the integration time  $T_{int}$  (control loop gain isn't unity, so must sample many times in order to converge)

*Flux* in a subaperture will increase with subap. area  $d^2$

# Integration time trades temporal error against measurement error

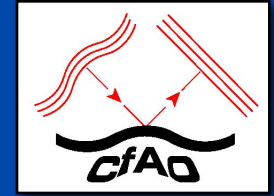


$$\sigma_p^2 \approx 6.3^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \left( \frac{10T_{int}}{\tau_0} \right)^{5/3}$$



From Hardy,  
Fig. 9.23

## First exercise in optimization: Choose optimum integration time



- Minimize the sum of read-noise and temporal errors by finding optimal integration time

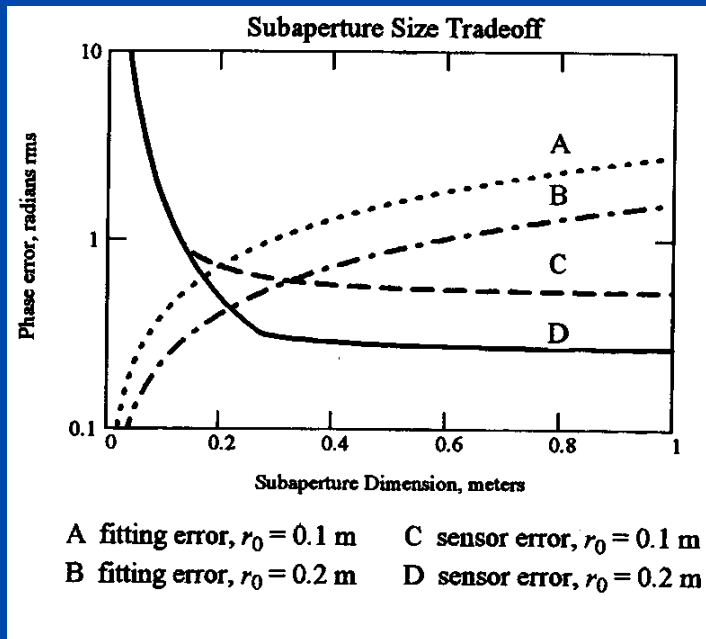
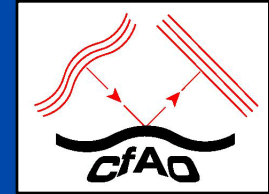
$$\sigma_p^2 = 6.3^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \left( \frac{10T_{int}}{\tau_0} \right)^{5/3}$$

$$\frac{d\sigma_p^2}{dT_{int}} = 0 = -2[6.3]^2 \frac{R^2 n_{pix}}{(Flux^2 T_{int}^3)} + \frac{5}{3} \left( \frac{10}{\tau_0} \right)^{5/3} T_{int}^{2/3}$$

$$T_{int}^{opt} = \left[ 1.03 \cdot \tau_0^{5/3} \frac{R^2 n_{pix}}{(Flux^2)} \right]^{3/11}$$

- Sanity check: optimum  $T_{int}$  larger for long  $\tau_0$ , larger read noise  $R$ , and lower photon  $Flux$

# Similarly, subaperture size $d$ trades fitting error against measurement error



$$\sigma_{\phi}^2 = 6.3^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \mu \left( \frac{d}{r_0} \right)^{5/3}$$

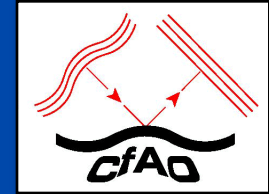
$Flux = I \times area = I \times d^2$  where intensity  $I = \text{photons}/(\text{sec cm}^2)$

$$\sigma_{\phi}^2 = 6.3^2 \frac{R^2 n_{pix}}{(I \times d^2 \times T_{int})^2} + \mu \left( \frac{d}{r_0} \right)^{5/3}$$

$$\sigma_{\phi}^2 = 6.3^2 \frac{R^2 n_{pix}}{(I \times T_{int})^2 d^2} + \mu \left( \frac{d}{r_0} \right)^{5/3}$$

Hardy, Figure 9.25

- Smaller  $d$ : better fitting error, worse measurement error

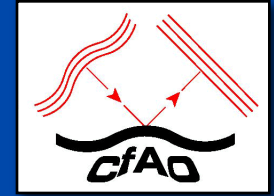


LGS (10th mag TT star) Case	Wavefront Error (nm)
Atmospheric Fitting Error	110
Bandwidth Error	146
High-order Measurement Error	150
LGS Focal Anisoplanatism Error	208
Uncorrectable Static Telescope Abs	66
Dyn WFS Zero-point Calib Error	80
Residual Na Layer Focus Change	36
High-Order Aliasing Error	37
Uncorrectable AO System Aberrations	30
Uncorrectable Instrum Aberrations	110
Angular Anisoplanatism Error	24
Tilt Measurement Error (one-axis)	11
Tilt Bandwidth Error (one-axis)	18
Residual Tel Pointing Jitter (1-axis)	96
Total Wavefront Error (nm) =	370
Strehl at K-band =	0.3
<b>Assumptions</b>	
Zenith angle (deg)	10
Guide star magnitude	10
HO WFS Rate (Hz)	500
TT Rate (Hz)	1000
Laser power (W)	16
WFS camera	CCD39
Subaperture diameter (m)	0.5625
Science Instrument	NIRC2
Amplitude of vibrations (arcsec)	0.2
d0 (m)	2.41
TT sensor	STRAP APDs

## Keck 2 AO error budget example

*(bright TT star)*





## Summary: What can you optimize when?

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- Once telescope is built on a particular site, you don't have control over  $\tau_0, \theta_0, r_0$
- But when you **build** your AO system, you CAN optimize choice of subaperture size  $d$ , maximum AO system speed, range of observing wavelengths, sky coverage, etc.
- Even when you are observing with an **existing** AO system, you can optimize:
  - wavelength of observations (changes fitting error)
  - integration time of wavefront sensor  $T_{int}$
  - tip-tilt bandwidth
  - brightness and angular offset of guide star